4.NF Comparing sums of unit fractions

Use <, =, or > to compare the following sums:

a. $\frac{1}{2} + \frac{1}{4}$ ———— $\frac{1}{3} + \frac{1}{5}$

b. $\frac{1}{3} + \frac{1}{2}$ ———— $\frac{1}{3} + \frac{1}{4}$

Commentary:
The purpose of this task is to help develop students' understanding of addition of fractions; it is intended as an instructional task. Notice that students are not asked to find the sum because in grade 4, students are limited to computing sums of fractions with the same denominator. Rather, they need to apply a firm understanding of unit fractions (fractions with one in the numerator) and reason about their relative size. That understanding begins with area models of fractions in grades one and two and expands to the number line in grade 3. With experience, students come to "know" that $\frac{1}{4}$ is less (smaller) than $\frac{1}{3}$ because dividing a whole into 4 rather than 3 pieces means there are more pieces, so they must be smaller.

To be successful with this type of problems, students must be able to easily identify the largest (or smallest) fraction in a group of unit fractions.

Struggling students can be given an easier version that repeats one of the fractions in both sums like:

$\frac{1}{2} + \frac{1}{-5}$———$\frac{1}{2} + \frac{1}{4}$

Students who can answer the original problem with ease can be asked about differences in problems like this:

$\frac{1}{2} - \frac{1}{-5}$———$\frac{1}{2} - \frac{1}{3}$

$\frac{1}{2} - \frac{1}{-5}$———$\frac{1}{3} - \frac{1}{5}$

Solution: Compare terms separately

a) Since $\frac{1}{2}$ is larger than $\frac{1}{3}$ and $\frac{1}{4}$ is larger than $\frac{1}{5}$ the sums satisfy: $\frac{1}{2} + \frac{1}{4} > \frac{1}{3} + \frac{1}{5}$

b) Since $\frac{1}{2}$ is larger than $\frac{1}{4}$, the sums satisfy: $\frac{1}{3} + \frac{1}{2} > \frac{1}{3} + \frac{1}{4}$

Solution: Compare the denominators

(a) By comparing the denominators I know that 1/2 is more then 1/3 because 2 is less then 3. I then compare 1/4 to 1/5 and know that 1/4 is more then 1/5 because 4 is less then 5.

(b) If you take 1/3 out of each equation you then need to only compare 1/2 to 1/4. I know that 1/2 is more then 1/4 because 2 is less then 4.

Solution: Compare visual representations

For both problems students can use the number line their teacher has provided as a scaffold.

(a) For this solution students may need four separate number lines. First a student marks of 1/2 and 1/4 on number lines and then 1/3 and 1/5 on two other number lines. They then put 1/2 and 1/4 side by side and visually compare it to 1/3 and 1/5 put side by side and see which set is longer.

(b) Students could remove the 1/3 from each side of the equation and then mark off 1/2 and 1/4 on two number lines and compare to see which was longer. Additionally they could use four number lines and mark off 1/2 and 1/3, 1/3 and 1/4 and compare that way.